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Massive Gravity in Curved Cosmological Backgrounds

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Abstract

We study the physical propagating modes in a massive gravity model in curved cosmological backgrounds, which we have found as classical solutions in our previous paper. We show that, generically, there exist such the cosmological background solutions consistent with the equations of motion where we assume the ghost condensation ansatzes. Using the (1+3)-parametrization of the metric fluctuations with 'unitary' gauge, we find that there is neither a scalar ghost nor a tachyon in the spectrum of the propagating modes, the tensor modes become massive owing to gravitational Higgs mechanism, and the model is free of the Boulware-Deser instability. The price we have to pay is that the scalar sector breaks the Lorentz-invariance, but there are no pathologies in the spectrum and lead to interesting phenomenology. Moreover, we present a proof of the absence of non-unitary modes for a specific ghost condensation model in a cosmological background.

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1 Introduction

In recent years, we have watched revival of interests to construct massive gravity theories from different physical motivations [1]-[16]. It concerns a very simple question: Can the graviton have a (small) mass consistent with experiments? If so, how?

One motivation behind this question comes from the astonishing observational fact that our universe is not just expanding but is at present in an epoch of undergoing an accelerating expansion [17, 18]. Although the standard model of cosmology is remarkably successful in accounting for many of observational facts of the universe, it is fair to say that we are still lacking a fundamental understanding of the late-time cosmic acceleration in addition to problems associated with dark matter and dark energy.

Several ideas have been thus far put forward for explaining this perplexing fact [19]. It is here that massive gravity theories might play an important role since massive gravity theories could modify Einstein's theory of general relativity at large cosmological scales and might lead to the present accelerated expansion of the universe without introducing still mysterious dark matter and dark energy at all. Since general relativity is almost the unique theory of massless spin 2 gravitational field whose universality class is determined by local symmetries under general coordinate transformations, any infrared modification of general relativity cannot aviod introduction of some kind of mass for the graviton.

The other motivation for attempting to construct massive gravity theories lies in string theory approach to quantum chromodynamics (QCD) [8]. For instance, as inspired in the large-N expansion of the gauge theory, which defines the planar (or genus zero) diagram and is analogous to the tree diagram of string theory, if we wish to apply a bosonic string theory to the gluonic sector in QCD, massless fields such as spin 2 graviton in string theory, must become massive or be removed somehow by an ingenious dynamical mechanism since such the massless fields do not appear in QCD. Note that this motivation is relevant to the modification of general relativity in the short distance region while the previous cosmological one is to the infrared modification. It is worthwhile to point out that the modification of both short and long distance regions reminds us of T-duality $R \leftrightarrow l_s^2/R$ in string theory where l_s is the fundamental string length scale. If there is the interesting symmetry behind the both distance regions in general relativity, such a modification could perhaps open a fruitful dialog between cosmology and QCD.

An approach for the construction of massive gravity theories is to take account of the spontaneous symmetry breakdown (SSB) of general coordinate reparametrization invariance [1]-[8]. Note that the SSB of general coordinate reparametrization invariance might provide a resolution for cosmological constant problem [6, 8]. Though the cosmological constant problem needs a theory of quantum gravity, solving the problem requires a low energy mechanism, for instance, a partial cancellation of vacuum energy density stemming from quantum loops. We expect that from the analogy with the Higgs mechanism in conventional gauge theories, the Nambu-Goldstone mode mixes with the massless graviton, thereby changing the vacuum structure of gravitational sector in a non-trivial way.

Recently, as a device for performing a consistent infrared modification of general relativity

an idea of ghost condensation has been proposed [9]. In this scenario, the '*unitary*' propagating scalar field appears as the Nambu-Goldstone boson for a spontaneously broken time-like diffeomorphism, and yields a possibility for resolving cosmological problems such as inflation, dark matter and dark energy. The key observation here is that a scalar ghost is converted to a normal scalar with positive-energy excitations.

More recently, 't Hooft has proposed a new Higgs mechanism for gravity where the massless graviton '*eats*' four real scalar fields and consequently becomes massive [8]. In his model, vacuum expectation values (VEV's) of the scalar fields are taken to be the four space-time coordinates by gauge-fixing diffeomorphisms, so the whole diffeomorphisms are broken spontanously. Of course, the number of dynamical degrees of freedom is left unchanged before and after the SSB. Actually, before the SSB of diffeomorphisms there are massless gravitons of two dynamical degrees of freedom and four real scalar fields whereas after the SSB we have massive gravitons of five dynamical degrees of freedom and one real scalar field. Afterward, a topological term was included to the 't Hooft model where an '*alternative*' metric tensor is naturally derived and the topological meaning of the gauge conditions was clarified [20]. [3]

The problem in the 't Hooft model is that a scalar field appearing after the SSB is a non-unitary propagating field so that in order to avoid violation of unitarity it must be removed from the physical Hibert space in terms of some procedure. [4] A resolution for this problem was offered where one requires the energy-momentum tensor of the matter field to couple to not the usual metric tensor but the modified metric one in such a way that the non-unitary scalar field does not couple to the energy-momentum tensor directly [8].

In this context, one could imagine that the ghost condensation scenario enables us to avoid emergence of the non-unitary scalar field in gravitational Higgs mechanism since the annoying non-unitary scalar field has its roots in one time-like component (i.e., ghost) in four scalar fields, to which the ghost condensation idea could be applied. Thus, it should be regarded that this observation is an alternative method for removing the non-unitary propagating scalar field in gravitational Higgs mechanism by 't Hooft.

In our previous article [11], as a first step for proving this conjecture, we have studied classical solutions in the unitary gauge in general ghost condensation models. This analysis is needed for understanding in what background gravitational Higgs mechanism arises. It turns out that depending on the form of scalar fields in an action, there are three kinds of classical, exact solutions, which are (anti-) de Sitter space-time, polynomially expanding universes and flat Minkowski space-time.

In this article, we wish to prove that there is indeed no non-unitary mode in the spectrum of propagating modes around cosmological backgrounds and the graviton becomes massive because of gravitational Higgs mechanism in these models. Thus, the models at hand are free of the problem associated with the non-unitary propagating mode and the Boulware-Deser instability [22] in curved backgrounds.

[3] Similar but different approaches have been already taken into consideration in Ref. [21].

[4] More recently, models of gravitational Higgs mechanism without the non-unitary propagating scalar field were proposed in Ref. [10].

This paper is organized as follows: In section 2, we review that there is an expanding cosmological solution with zero acceleration to the equations of motion in a massive gravity model, what we call, 't Hooft model with ghost condensation. In section 3, we consider a more general model and look for classical solutions where we will find expanding cosmological solutions with non-zero acceleration. In section 4, based on the massive gravity model in section 3, the propagating modes around the cosmological backgrounds are examined in detail by using the (1+3)-parametrization of the metric fluctuations. We find that the tensor modes become massive, the vector modes are non-dynamical, three of the scalar modes are also non-dynamical and one scalar becomes massive. This scalar is originally non-unitary mode, that is, a ghost, but becomes a normal particle because of the ghost condensation mechanism. However, the dispersion relation is not usual one, so this mode breaks the Lorentz invariance as expected from the ghost condensation scenario. In section 5, we present a proof of the absence of non-unitary modes for the specific ghost condensation model treated in section 2. The final section is devoted to conclusions and discussion.

2 Cosmological solution in 't Hooft model with ghost condensation

In this section, we wish to review our previous work [11] and explain how to derive classical solutions in massive gravity models.

Let us start with review of our 't Hooft model with ghost condensation. The general proof that there is no non-unitary scalar ghost in this model will be given in section 5. The action of ghost condensation [9], which is inspired by gravitational Higgs mechansim, takes the form [5]:

$$\begin{aligned} S &= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}[R-2\Lambda+f(X)-g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b\eta_{ab}] \\ &= \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}[R-2\Lambda+F(X)-g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j\delta_{ij}], \qquad (1) \end{aligned}$$

where η_{ab} is the internal flat metric with diagonal elements $(-1,+1,+1,+1)$. The indices $\mu,\nu(=0,1,2,3)$ and $i,j(=1,2,3)$ are space-time and spatial indices, respectively. And we have defined $F(X)\equiv X+f(X)$ where $f(X)$ is a function of X which is defined as

$$X = g^{\mu\nu}\partial_\mu\phi^0\partial_\nu\phi^0. \qquad (2)$$

Let us note that the last term in the first equality in (1) is added to the original ghost condensation action, or alternatively speaking from the side of gravitational Higgs mechanism, the third term $f(X)$ is introduced to the original action of gravitational Higgs mechanism.

[5]For simplicity, we have put the Newton constant G_N at the front, so the dimension of ghost-like scalar field ϕ^0 differs from that of the original ghost condensation model, but it is easy to modify the dimension by field redefinition.

Recall that the ghost condensation scenario consists of three ansatzes: The first ansatz amounts to requiring that the function $F(X)$ has a minimum at some point X_{min} such that

$$F_X(X_{min}) = 0, \ F_{XX}(X_{min}) > 0, \tag{3}$$

where $F_X(X_{min})$, for instance, means the differentiation of $F(X)$ with respect to X and then putting $X = X_{min}$. Note that the latter condition in (3) ensures ghost condensation.

As the second ansatz, the ghost-like scalar field ϕ^0 is expanded around the background mt as

$$\phi^0 = mt + \pi, \tag{4}$$

where π is the small fluctuation. This equation can be interpreted as follows: Using a time-like diffeomorphism $\delta\phi^0 = \varepsilon^0$, one can take the gauge $\pi = 0$. In other words, π is a Nambu-Goldstone boson associated with spontaneous symmetry breakdown of time traslation.

The last ansatz is natural from the second ansatz. That is, since time coodinate t plays a distinct role from spatial coordinates $x^i (i = 1, 2, 3)$, it is plausible to make an assumption on background space-time metric, which is of the Friedmann-Robertson-Walker form

$$ds^2 = -dt^2 + a(t)^2 d\Omega^2, \tag{5}$$

where $a(t)$ is the scale factor and $d\Omega^2$ is the spatial metric for a maximally symmetric three-dimensional space.

The equations of motion are easily derived as follows:

$$\begin{aligned} \partial_\mu(\sqrt{-g} g^{\mu\nu} \partial_\nu \phi^i) &= 0, \\ \partial_\mu(\sqrt{-g} g^{\mu\nu} F_X(X) \partial_\nu \phi^0) &= 0, \\ G_{\mu\nu} + \Lambda g_{\mu\nu} &= T_{\mu\nu}, \end{aligned} \tag{6}$$

where the stress-energy tensor is defined as

$$\begin{aligned} T_{\mu\nu} &= (\partial_\mu \phi^a \partial_\nu \phi^b - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \phi^a \partial_\beta \phi^b) \eta_{ab} \\ &- (\partial_\mu \phi^0 \partial_\nu \phi^0 f_X(X) - \frac{1}{2} g_{\mu\nu} f(X)). \end{aligned} \tag{7}$$

Now let us solve these equations of motion under the ansatzes of ghost condensation and the *unitary* gauge for diffeomorphisms

$$\phi^a = m x^\mu \delta^a_\mu. \tag{8}$$

It is easy to check that the ϕ^i-equations are trivially satisfied. Moreover, the ϕ^0-equation reduces to the expression

$$\partial_0(a(t)^3 F_X(X)) = 0, \tag{9}$$

whose validity requires us that one of ghost condensation ansatzes should be automatically satisfied, i.e., $F_X(X_{min}) = 0$ since $X_{min} \equiv X(\phi^0 = mt) = -m^2$ and $\partial_0 X_{min} = 0$.

Finally, Einstein's equations are cast to

$$\begin{aligned} 3(\frac{\dot{a}}{a})^2 - \tilde{\Lambda} &= \frac{3m^2}{2a^2}, \\ -2\frac{\ddot{a}}{a} - (\frac{\dot{a}}{a})^2 + \tilde{\Lambda} &= -\frac{m^2}{2a^2}, \end{aligned} \tag{10}$$

where we have defined $\tilde{\Lambda} = \Lambda - \frac{1}{2}F(-m^2)$ and the dot over $a(t)$ denotes the derivative with respect to t. Here we have made use of the expression of the stress-energy tensor which is obtained by inserting Eq. (8) to Eq. (7)

$$T_{\mu\nu} = m^2(\eta_{\mu\nu} - \frac{1}{2}g_{\mu\nu}g^{ab}\eta_{ab}) + m^2\delta^0_\mu\delta^0_\nu + \frac{1}{2}g_{\mu\nu}f(-m^2). \tag{11}$$

From the equations (10), we can eliminate the terms involving $\dot{a}$ whose result is written as

$$3\frac{\ddot{a}}{a} = \tilde{\Lambda}. \tag{12}$$

It then turns out that the general solution for this equation exists only when $\tilde{\Lambda} = 0$. Thus, Eq. (12) reads $\ddot{a} = 0$, so we have the general solution

$$a(t) = c(t - t_0), \tag{13}$$

where c and t_0 are integration constants. Substituting this solution into the first equation in Eq. (10), c is fixed to $c = \pm\frac{m}{\sqrt{2}}$. As a result, the line element takes the form

$$ds^2 = -dt^2 + \frac{m^2}{2}(t - t_0)^2 d\Omega^2, \tag{14}$$

which describes the linearly expanding universe with zero acceleration. [6] Notice that this solution is a unique classical solution, so a flat Minkowski space-time, for instance, is not a solution of this model. Here it is worthwhile to mention that although the present universe seems to be accelerating so that the linearly expanding universe is excluded from WMAP experiment [23], the general solution (14) might be useful in describing the future or past status of universe.

3 Cosmological solutions in more general massive gravity models

In the previous section, we have found an interesting cosmological solution, which is very similar to the Milne universe, in the 't Hooft model with ghost condensation. However, the

[6] This solution was also obtained in a different massive gravity model [10].

solution has zero acceleration so that it does not describe the present epoch of the accelerating universe. It would be more desirable if we could get a classical solution with non-zero acceleration. It has been shown in our previous paper [11] that this desire is actually realized if one generalizes the model in section 2 to, what is called, a general ghost condensation model. In this article, instead of explaining the derivation of the solutions in the general ghost condensation model, we show that the solutions also exist in more general massive gravity models with the ghost condensation potential.

In this section, we will obey the following line of arguments: In order to make the analysis of propagating modes easier, which will be done in the next section, we will first choose a simple massive gravity model in a general class of those models. Next, using this specific model, we will show that there are polynomially expanding universes' solutions with non-zero acceleration. Finally, we will examine under what conditions the solutions at hand remain the solutions to the equations of motion in the most general models.

Now we shall take a different line element from (5) since the form is more convenient for later analysis in section 4. Of course, at a final stage it is easy to transform to the expression like (5) through a global coordinate transformation. The form we take as the line element is of conformal flat type:

$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= e^{2A(t)} \eta_{\mu\nu} dx^\mu dx^\nu, \end{aligned} \tag{15}$$

where $\eta_{\mu\nu}$ is the flat metric.

We consider a simple massive gravity model [7]:

$$\begin{aligned} S &= \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} [R - 2\Lambda + F(X) - G(W^{ij})] \\ &\equiv \frac{1}{16\pi G_N} \int d^4x [L_{EH} + L_\Lambda + L_F + L_G], \end{aligned} \tag{16}$$

where X is defined in (2) and W^{ij} is defined via new variables Y^{ij} and V^i as

$$\begin{aligned} W^{ij} &= Y^{ij} - \frac{V^i V^j}{X}, \\ Y^{ij} &= g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^j, \\ V^i &= g^{\mu\nu} \partial_\mu \phi^0 \partial_\nu \phi^i. \end{aligned} \tag{17}$$

This model of massive gravity was obtained by considering the residual diffeomorphisms

$$x^i \to x'^i = x^i + \zeta^i(t), \tag{18}$$

[7]For the analogy with the previous model in section 2, we have introduced two different functions $F(X)$ and $G(W^{ij})$ separately for variables X and W^{ij}, but it is more economical to do only one function $F(X, W^{ij})$ dependent on two variables. This simple model has been investigated in Refs. [14]-[16].

which are translated to the symmetries of the scalar fields as

$$\phi^i \to \phi'^i = \phi^i + \xi^i(\phi^0), \tag{19}$$

where $\zeta^i(t)$ and $\xi^i(\phi^0)$ are the infinitesimal transformation parameters which are dependent on t and ϕ^0, respectively. Note that both X and W^{ij} are invariant under (19).

Let us show that there are polynomially expanding cosmological solutions with non-zero acceleration in this model. To this end, we first derive the equations of motion. The ϕ^i-, ϕ^0-, and Einstein's equations are derived in a straightforward manner

$$\begin{aligned}
\partial_\mu[\sqrt{-g}g^{\mu\nu}\frac{\partial G}{\partial W^{ij}}(\partial_\nu\phi^j - \frac{V^j}{X}\partial_\nu\phi^0)] &= 0, \\
\partial_\mu[\sqrt{-g}g^{\mu\nu}\{\partial_\nu\phi^0 F_X + (\frac{V^j}{X}\partial_\nu\phi^i - \frac{V^iV^j}{X^2}\partial_\nu\phi^0)\frac{\partial G}{\partial W^{ij}}\}] &= 0, \\
G_{\mu\nu} + \Lambda g_{\mu\nu} &= T_{\mu\nu},
\end{aligned} \tag{20}$$

with the stress-energy tensor being given by

$$\begin{aligned}
T_{\mu\nu} &= -\partial_\mu\phi^0\partial_\nu\phi^0 F_X + [\partial_\mu\phi^i\partial_\nu\phi^j - \frac{2}{X}\partial_\mu\phi^0\partial_\nu\phi^i V^j + \frac{V^iV^j}{X^2}\partial_\mu\phi^0\partial_\nu\phi^0]\frac{\partial G}{\partial W^{ij}} \\
&+ \frac{1}{2}g_{\mu\nu}[F - G].
\end{aligned} \tag{21}$$

With the *unitary* gauge (8), each variable takes the form

$$\begin{aligned}
X &= m^2 g^{00} = -m^2 e^{-2A} \equiv \bar{X}, \\
V^i &= m^2 g^{0i} = 0 \equiv \bar{V}^i, \\
Y^{ij} &= m^2 g^{ij} = m^2 e^{-2A}\delta_{ij} \equiv \bar{Y}^{ij}, \\
W^{ij} &= m^2 e^{-2A}\delta_{ij} \equiv \bar{W}\delta_{ij} \equiv \bar{W}^{ij}.
\end{aligned} \tag{22}$$

Then, the ϕ^i-equations are reduced to

$$\partial_i[e^{2A}\frac{\partial G(\bar{W}^{ij})}{\partial W^{ij}}] = 0, \tag{23}$$

which is trivially satisfied since the quantity in the square bracket depends on only t.

Next, the ϕ^0-equation reads

$$\partial_t[e^{2A}F_X(\bar{X})] = 0, \tag{24}$$

which is satisfied by one of ghost condensation ansatzes, $F_X(\bar{X}) = 0$. Furthermore, Einstein's equations are recast to

$$\begin{aligned}
&\ddot{A}(-2\delta^0_\mu\delta^0_\nu - 2\eta_{\mu\nu}) + (\dot{A})^2(2\delta^0_\mu\delta^0_\nu - \eta_{\mu\nu}) + \Lambda e^{2A}\eta_{\mu\nu} \\
&= m^2\delta^i_\mu\delta^j_\nu\frac{\partial G(\bar{W}^{ij})}{\partial W^{ij}} + \frac{1}{2}e^{2A}\eta_{\mu\nu}[F(\bar{X}) - G(\bar{W}^{ij})].
\end{aligned} \tag{25}$$

In order to proceed further, we need to fix the form of the potential term $G(W^{ij})$. It then turns out that there are non-trivial cosmological solutions provided that we select

$$G(W^{ij}) = \frac{1}{3} K Tr(W^{ij})^n - 2\tilde{\Lambda}, \tag{26}$$

where K is a positive constant and $\tilde{\Lambda} \equiv \Lambda - \frac{1}{2}F(\bar{X})$. For later convenience, using this expression (26) let us define the following two quantities

$$\begin{aligned} \frac{\partial G(\bar{W}^{ij})}{\partial W^{ij}} &= G_1 \delta_{ij}, \\ \frac{\partial^2 G(\bar{W}^{ij})}{\partial W^{ij} \partial W^{kl}} &= G_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}), \end{aligned} \tag{27}$$

where G_1 and G_2 are defined as

$$\begin{aligned} G_1 &= \frac{K}{3} n \bar{W}^{n-1} = \frac{K}{3} n (m^2 e^{-2A})^{n-1}, \\ G_2 &= \frac{K}{3} \frac{n(n-1)}{2} \bar{W}^{n-2} = \frac{K}{3} \frac{n(n-1)}{2} (m^2 e^{-2A})^{n-2}. \end{aligned} \tag{28}$$

As a result, Einstein's equations read

$$\begin{aligned} (\dot{A})^2 &= -\frac{1}{6}[F(\bar{X}) - G(\bar{W}^{ij}) - 2\Lambda] e^{2A}, \\ \ddot{A} + \frac{1}{2}(\dot{A})^2 &= -\frac{1}{4}[F(\bar{X}) - G(\bar{W}^{ij}) - 2\Lambda] e^{2A} - \frac{1}{2} m^2 G_1. \end{aligned} \tag{29}$$

These equations are easily integrated to be

$$e^A = [\pm\sqrt{\frac{K}{6}}(n-1) m^n (t - t_0)]^{\frac{1}{n-1}}, \tag{30}$$

where t_0 is an integration constant. Then, the line element becomes (after a suitable redefinition of x^μ by an overall constant factor)

$$\begin{aligned} ds^2 &= (t - t_0)^{\frac{2}{n-1}} \eta_{\mu\nu} dx^\mu dx^\nu \\ &= -d\tau^2 + (\tau - \tau_0)^{\frac{2}{n}} dx^i dx_i, \end{aligned} \tag{31}$$

where the latter expression informs us that the second derivative of the scale factor, that is, the acceleration, is positive for $n < 1$. Of course, this condition might not be meaningful phenomenologically since the present universe seems to be entering a new era of *exponentially* accelerating inflation again, which is controled by the equation of state $w = \frac{P}{\rho} = -1$, whereas our solutions describe the *polynomially* accelerating universes. Nevertheless, we shall take account of this condition since at present there do not exist sufficient evidences to exclude the *polynomially* accelerating universe such as quintessence ($-1 < w < -\frac{1}{3}$) and phantom ($w < -1$) etc.

In this way, we have obtained an interesting class of cosmological solutions, which have a behavior of polynomially expanding universes with non-zero acceleration, by generalizing a massive gravity model in such a way that the potential terms in the action include functions of not only ϕ^0 but also ϕ^i.

To close this section, it is valuable to ask ourselves whether such the solutions exist even in the most general models or not. Recall that the symmetries (19) constrain the model to some degree in the sense that the potential terms are not a general function of X, V^i and Y^{ij} but the more restricted function of X and W^{ij} which are invariant under (19).

We therefore take the potential term to be the most general function, which is an arbitrary function of X, V^i and Y^{ij}:

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}[R - 2\Lambda + U(X, V^i, Y^{ij})]. \tag{32}$$

Now let us derive the equations of motion under the unitary gauge (8) whose concrete expressions are given by

$$\begin{aligned} \partial_\mu[\sqrt{-g}(\frac{1}{2}g^{\mu 0}\frac{\partial U}{\partial V^i} + g^{\mu j}\frac{\partial U}{\partial Y^{ij}})] &= 0, \\ \partial_\mu[\sqrt{-g}(g^{\mu 0}\frac{\partial U}{\partial X} + \frac{1}{2}g^{\mu i}\frac{\partial U}{\partial V^i})] &= 0, \\ G_{\mu\nu} + \Lambda g_{\mu\nu} &= T_{\mu\nu}. \end{aligned} \tag{33}$$

Now, the second ϕ^0-equation is satisfied if one imposes the ghost condensation ansatz

$$\frac{\partial U(\bar{X}, \bar{V}^i, \bar{Y}^{ij})}{\partial X} = 0. \tag{34}$$

Next, we find that the first ϕ^i-equations are satisfied if and only if there exist new conditions on U

$$\frac{\partial U(\bar{X}, \bar{V}^i, \bar{Y}^{ij})}{\partial V^i} = 0. \tag{35}$$

In this regard, note that in order to keep the $SO(3)$ rotational symmetry, U must be a function of V^iV^i, $V^iY_{ij}V^j$ etc., so $\frac{\partial U(\bar{V}^i)}{\partial V^i}$ would be proportional to $\bar{V}^i$. In the unitary gauge, $\bar{V}^i$ is vanishing so that the conditions (35) are valid.

The remaining equations of motion are Einstein's equations, which read in the unitary gauge

$$\begin{aligned} &\ddot{A}(-2\delta_\mu^0\delta_\nu^0 - 2\eta_{\mu\nu}) + (\dot{A})^2(2\delta_\mu^0\delta_\nu^0 - \eta_{\mu\nu}) + \Lambda e^{2A}\eta_{\mu\nu} \\ = &\frac{1}{2}e^{2A}\eta_{\mu\nu}U - m^2(\delta_\mu^0\delta_\nu^0\frac{\partial U}{\partial X} + \delta_\mu^0\delta_\nu^i\frac{\partial U}{\partial V^i} + \delta_\mu^i\delta_\nu^j\frac{\partial U}{\partial Y^{ij}}). \end{aligned} \tag{36}$$

The requirement that these equations should have the same solutions as (30) amounts to the similar conditions to (26) and (28), which are

$$\begin{aligned} U(\bar{X}, \bar{V}^i, \bar{Y}^{ij}) &= -K(m^2e^{-2A})^n + 2\Lambda, \\ \frac{\partial U(\bar{X}, \bar{V}^i, \bar{Y}^{ij})}{\partial Y^{ij}} &= -\frac{K}{3}n(m^2e^{-2A})^{n-1}\delta_{ij}. \end{aligned} \tag{37}$$

These conditions appear in a natural way when we assume the potential term U to be the polynomial type like (26), so we could conclude that the classical solutions (30) exist as well in the most general massive gravity models by picking up an appropriate form of the ghost potential.

4 Analysis of propagating modes

In this section, we analyse the physical modes propagating in the cosmological backgrounds obtained in section 3. Thus, let us consider small fluctuations around the metric

$$g_{\mu\nu} = e^{2A(t)}(\eta_{\mu\nu} + h_{\mu\nu}). \tag{38}$$

With Eq. (38), some quantities relevant to the metric tensor read in the second-order approximation level of the fluctuations $h_{\mu\nu}$

$$\begin{aligned} g^{\mu\nu} &= e^{-2A(t)}(\eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\alpha}h_\alpha^\nu), \\ \sqrt{-g} &= e^{4A(t)}(1 + \frac{1}{2}h - \frac{1}{4}h_{\mu\nu}h^{\mu\nu} + \frac{1}{8}h^2), \\ \sqrt{-g}g^{\mu\nu} &= e^{2A(t)}[\eta^{\mu\nu} - h^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu}h + (-\frac{1}{4}h_{\alpha\beta}h^{\alpha\beta} + \frac{1}{8}h^2)\eta^{\mu\nu} \\ &- \frac{1}{2}hh^{\mu\nu} + h^{\mu\alpha}h_\alpha^\nu], \end{aligned} \tag{39}$$

etc. Here, summation over space-time indices $\mu = 0, 1, 2, 3$ is carried out with the flat Minkowski metric $\eta_{\mu\nu}$ while that over spatial indices $i = 1, 2, 3$ is done with the Kronecker δ_{ij}, so we have, for instance,

$$h = \eta_{\mu\nu}h^{\mu\nu} = -h_{00} + h_{ii}. \tag{40}$$

In the analysis of the physical modes, it is convenient to make use of the $(1 + 3)$-parametrization of the metric fluctuations [24]

$$\begin{aligned} h_{00} &= 2\phi, \\ h_{0i} &= S_i + \partial_i B, \\ h_{ij} &= h_{ij}^{TT} - (\partial_i F_j + \partial_j F_i) - 2(\psi\delta_{ij} - \partial_i\partial_j E), \end{aligned} \tag{41}$$

where h_{ij}^{TT} is a traceless, transverse spatial tensor

$$\partial_i h_{ij}^{TT} = 0 = h_{ii}^{TT}, \tag{42}$$

and S_i and F_i are transverse spatial vectors

$$\partial_i S_i = 0 = \partial_i F_i. \tag{43}$$

In this parametrization, the tensor h_{ij}^{TT} has 2 degrees of freedom (d.o.f.), the vectors S_i and F_i do $2 \times 2 = 4$ d.o.f., and the scalars ϕ, B, ψ and E do $1 \times 4 = 4$ d.o.f., so in total 10 d.o.f. which is exactly equal to the number of $h_{\mu\nu}$.

Incidentally, recall that in Einstein's general relativity only the massless spin 2 graviton h_{ij}^{TT} of 2 d.o.f. is physical owing to general coordinate invariance. On the other hand, in a general massive gravity model [8], the spin 2 tensor h_{ij}^{TT} of 2 d.o.f., the spin 1 vector F_i of 2 d.o.f. and the spin 1 scalar ψ (or E) of 1 d.o.f. are physical so totally we have the massive graviton of 5 d.o.f. (the other modes are non-dynamical) which involves spins $\pm 2, \pm 1$ and 0. The important point to notice is that there in general remains one scalar mode E (or ψ), which has negative norm, so it is in essence a *ghost*! This peculiar feature, that is, the existence of a scalar ghost in the physical Hilbert space, is usually avoided by removing the ghost with the help of an enhancement of gauge symmetry as in the Fierz-Pauli Lorentz-covariant massive gravity model [25]. However, this nice property is lost in curved backgrounds owing to the disappearance of the enhanced gauge symmetry, and in consequence we have the Boulware-Deser instability scalar mode [22].

Accordingly, the real problem is to find a method of removal of the ghost mode from the physical spectrum without recourse to the enhanced gauge symmetry. In this article, we try to remove the ghost by appealing to the ghost condensation mechanism.

After a little lenghty calculation, the quadratic part of the Einstein-Hilbert action and the cosmological term in (16) is decomposed into tensor, vector and scalar sectors up to total derivative terms

$$\begin{aligned} L_{EH}^{(2)} &= L_{EH}^{(T)} + L_{EH}^{(V)} + L_{EH}^{(S)}, \\ L_{\Lambda}^{(2)} &= L_{\Lambda}^{(T)} + L_{\Lambda}^{(V)} + L_{\Lambda}^{(S)}, \end{aligned} \tag{44}$$

where each term is given by

$$\begin{aligned} L_{EH}^{(T)} &= e^{2A}[-\{\ddot{A} + \frac{1}{2}(\dot{A})^2\}(h_{ij}^{TT})^2 + \frac{1}{4}\{(\partial_0 h_{ij}^{TT})^2 - (\partial_i h_{jk}^{TT})^2\}], \\ L_{EH}^{(V)} &= e^{2A}[3(\dot{A})^2 S_i^2 + 2\{\ddot{A} + \frac{1}{2}(\dot{A})^2\}F_i \Delta F_i - \frac{1}{2}(S_i + \partial_0 F_i)\Delta(S_i + \partial_0 F_i)], \\ L_{EH}^{(S)} &= e^{2A}[(\dot{A})^2\{-9\phi^2 - 9\psi^2 + 2(9\phi\psi - 3\phi\Delta E + \frac{3}{2}(\partial_i B)^2) - 2(\psi\Delta E + \frac{1}{2}(\Delta E)^2)\} \\ &+ \dot{A}(4\phi\Delta B + 12\phi\partial_0\psi - 4\phi\partial_0\Delta E) - 4\ddot{A}(\psi\Delta E + \frac{1}{2}(\Delta E)^2) \\ &- 4\partial_0\psi\Delta B - 4\phi\Delta\psi + 6\psi\partial_0^2\psi - 2\psi\Delta\psi + 4\partial_0\psi\partial_0\Delta E], \end{aligned} \tag{45}$$

and

$$\begin{aligned} L_{\Lambda}^{(T)} &= \frac{1}{2}\Lambda e^{4A}(h_{ij}^{TT})^2, \\ L_{\Lambda}^{(V)} &= -\Lambda e^{4A}(S_i^2 + F_i \Delta F_i), \\ L_{\Lambda}^{(S)} &= -2\Lambda e^{4A}[-\frac{1}{2}\phi^2 + 3\phi\psi - \phi\Delta E + \frac{3}{2}\psi^2 - \psi\Delta E - \frac{1}{2}(\Delta E)^2 + \frac{1}{2}(\partial_i B)^2], \end{aligned} \tag{46}$$

[8] Here the term *general* means that there is no special residual gauge symmetry.

where Δ denotes the Laplacian operator defined as $\Delta \equiv \partial_i^2$.

Furthermore, provided that by L_{F+G} we denote the sum of the quadratic part of the potential terms L_F and L_G, it is also decomposed into each spin sector as

$$\begin{aligned}
L_{F+G}^{(T)} &= [-\frac{1}{4}e^{4A}(F-G) - e^{2A}m^2G_1 - m^4G_2](h_{ij}^{TT})^2, \\
L_{F+G}^{(V)} &= [\frac{1}{2}e^{4A}(F-G) + 2e^{2A}m^2G_1 + 2m^4G_2]F_i\Delta F_i + \frac{1}{2}e^{4A}(F-G)S_i^2, \\
L_{F+G}^{(S)} &= e^{4A}[-\frac{1}{2}(F-G) + 2F_{XX}\bar{X}^2]\phi^2 + \frac{1}{2}e^{4A}(F-G)(\partial_i B)^2 \\
&+ [e^{4A}(F-G) + 2e^{2A}m^2G_1]\phi(3\psi - \Delta E) + [\frac{3}{2}e^{4A}(F-G) + 6e^{2A}m^2G_1 - 12m^4G_2]\psi^2 \\
&+ [-e^{4A}(F-G) - 4e^{2A}m^2G_1 + 8m^4G_2]\psi\Delta E \\
&+ [-\frac{1}{2}e^{4A}(F-G) - 2e^{2A}m^2G_1 - 4m^4G_2](\Delta E)^2, \qquad (47)
\end{aligned}$$

where F, F_{XX} and G denote $F(\bar{X})$, $F_{XX}(\bar{X})$ and $G(\bar{W}^{ij})$, respectively.

We are now in a position to discuss each spin sector in order. In the tensor sector, the total Lagrangian takes the form

$$\begin{aligned}
L^{(T)} &\equiv L_{EH}^{(T)} + L_{\Lambda}^{(T)} + L_{F+G}^{(T)} \\
&= \frac{1}{4}e^{2A}[(\partial_0 h_{ij}^{TT})^2 - (\partial_i h_{jk}^{TT})^2] + [-\frac{1}{2}e^{2A}m^2G_1 - m^4G_2](h_{ij}^{TT})^2, \qquad (48)
\end{aligned}$$

where the background equations, those are, Einstein's equations (29) are used to simplify the expression. Then, the equations of motion for h_{ij}^{TT} give

$$\Box h_{ij}^{TT} - 2\dot{A}\partial_0 h_{ij}^{TT} - 2(m^2G_1 + 2m^4G_2e^{-2A})h_{ij}^{TT} = 0, \qquad (49)$$

where the d'Alembertian operator is defined as $\Box = \partial_\mu\partial^\mu = -\partial_0^2 + \Delta$.

In order to see what mass of the graviton is, let us introduce

$$\tilde{h}_{ij} = e^{A(t)}h_{ij}^{TT}. \qquad (50)$$

In terms of $\tilde{h}_{ij}$, the equations of motion (49) read

$$\Box\tilde{h}_{ij} + [\ddot{A} + (\dot{A})^2 - 2(m^2G_1 + 2m^4G_2e^{-2A})]\tilde{h}_{ij} = 0. \qquad (51)$$

Thus, the effective mass square of the tensor modes, M_h^2 is given by

$$\begin{aligned}
M_h^2 &= -[\ddot{A} + (\dot{A})^2 - 2(m^2G_1 + 2m^4G_2e^{-2A})] \\
&= \frac{4n^2+n-2}{(n-1)^2}\frac{1}{(t-t_0)^2}, \qquad (52)
\end{aligned}$$

where we have used (30) and (28) in the second equality.

Thus, the tensor modes in the cosmological backgrounds are massive for $n > \frac{-1+\sqrt{33}}{16}$ or $n < \frac{-1-\sqrt{33}}{16}$ $(n \neq 1)$ while for $\frac{-1-\sqrt{33}}{16} < n < \frac{-1+\sqrt{33}}{16}$ they are effectively tachyonic and the backgrounds become unstable at least perturbatively. For $n = \frac{-1\pm\sqrt{33}}{16}$, the graviton becomes massless. Moreover, the effective mass of the graviton approaches zero in the limit $t \to \infty$.

We next move to the vector sector. With the help of Einstein's equations (29), the total Lagrangian for vector modes can be written as

$$\begin{aligned} L^{(V)} &\equiv L^{(V)}_{EH} + L^{(V)}_{\Lambda} + L^{(V)}_{F+G} \\ &= -\frac{1}{2} e^{2A}(S_i + \partial_0 F_i)\Delta(S_i + \partial_0 F_i) + (e^{2A} m^2 G_1 + 2m^4 G_2) F_i \Delta F_i. \end{aligned} \tag{53}$$

Taking the variation with respect to S_i, we have the equation

$$S_i = -\partial_0 F_i. \tag{54}$$

Substituting it into $L^{(V)}$, we have only the second term in (53)

$$L^{(V)} = (e^{2A} m^2 G_1 + 2m^4 G_2) F_i \Delta F_i. \tag{55}$$

The equations of motion for F_i therefore yield

$$F_i = 0. \tag{56}$$

Plugging it back into $L^{(V)}$ in (55) again, the Lagrangian becomes identically vanishing, so that F_i do not obey any equations of motion and take any values.

This arbitrariness, of course, is a consequence of the residual diffeomorphism invariance (18). In this case, the reparametrization symmetries read

$$\delta h_{\mu\nu} = 2\dot{A}\eta_{\mu\nu}\zeta^0 + \partial_\mu \zeta_\nu + \partial_\nu \zeta_\mu. \tag{57}$$

With $\zeta_i = \zeta_i(t)$ in (18) and the parametrization (41), we have the residual gauge symmetries

$$\delta S_i = \partial_0 \zeta_i, \ \delta F_i = -\zeta_i, \tag{58}$$

from which the modes F_i and S_i become non-dynamical. More precisely speaking, it is F_i that the residual gauge symmetries (58) make non-dynamical since S_i remain non-dynamical irrespective of the existence of mass term as easily seen in (53). In this way, we have proved that all the vector modes are not physical but non-dynamical in this simple massive gravity model.

Finally, we are ready to examine the scalar sector, which is known to be the most problematic and complicated. In fact, the Boulware-Deser mode [22] appears in this sector in the Fierz-Pauli massive gravity model [25].

After utilizing Einstein's equations (29), the total Lagrangian is of form

$$
\begin{aligned}
L^{(S)} &\equiv L_{EH}^{(S)} + L_{\Lambda}^{(S)} + L_{F+G}^{(S)} \\
&= 2e^{2A}[-2\phi\Delta\psi - 2\partial_0\psi\Delta B + 2\partial_0\psi\partial_0\Delta E + 3\psi\partial_0^2\psi - \psi\Delta\psi \\
&+ \dot{A}(2\phi\Delta B - 2\phi\partial_0\Delta E + 6\phi\partial_0\psi)] + e^{4A}(F - G - 2\Lambda + 2F_{XX}\bar{X}^2)\phi^2 \\
&+ 3[e^{4A}(F - G - 2\Lambda) + 2e^{2A}m^2G_1 - 4m^4G_2]\psi^2 + 2m^2G_1e^{2A}(3\phi\psi - \phi\Delta E) \\
&- 2m^2G_1e^{2A}[\psi\Delta E + \frac{1}{2}(\Delta E)^2] + 8m^4G_2[\psi\Delta E - \frac{1}{2}(\Delta E)^2].
\end{aligned} \tag{59}
$$

The equation of motion for B gives

$$
\partial_0\psi = \dot{A}\phi. \tag{60}
$$

Thus, integrating over B and ϕ and using Einstein's equations again, up to total surface terms $L^{(S)}$ is reduced to

$$
\begin{aligned}
L^{(S)} &= \frac{2}{(\dot{A})^2}m^4F_{XX}\partial_0\psi\partial_0\psi + 2ne^{2A}\psi\Delta\psi - 3nm^2e^{2A}G_1\psi^2 \\
&- \frac{2}{\dot{A}}e^{2A}m^2G_1\partial_0\psi\Delta E - 2e^{2A}m^2G_1[\psi\Delta E + \frac{1}{2}(\Delta E)^2] \\
&+ 8m^4G_2[\psi\Delta E - \frac{1}{2}(\Delta E)^2].
\end{aligned} \tag{61}
$$

Here the key point to understanding a scalar *ghost* is that there is no more $\partial_0\psi\partial_0\Delta E$ which is cancelled when integrating over B and ϕ (see Eqs. (60) and (59)). Furthermore, the scalar mode E turns out to be non-dynamical since there is no time-derivative term of E, so we can integrate over the mode E. Consequently, $L^{(S)}$ becomes a Lagrangian for only a remaining scalar mode ψ. A short calculation shows that

$$
\begin{aligned}
L^{(S)} &= \frac{2}{(\dot{A})^2}[m^4F_{XX} + \frac{n}{2n-1}\frac{K}{6}m^{2n}e^{-2(n-2)A}]\partial_0\psi\partial_0\psi \\
&+ 2ne^{2A}\psi\Delta\psi - \frac{n^3}{2n-1}\frac{4K}{3}m^{2n}e^{-2(n-2)A}\psi^2.
\end{aligned} \tag{62}
$$

This Lagrangian gives us several important information on ψ. First, as far as ghost is concerned, the scalar mode ψ never be a ghost when $I \equiv \frac{2}{(\dot{A})^2}[m^4F_{XX} + \frac{n}{2n-1}\frac{K}{6}m^{2n}e^{-2(n-2)A}] > 0$. Recall that $F_{XX} > 0$ from one of the ghost condensation ansatzes (3), so I is definitely positive as long as $n > \frac{1}{2}$. Moreover, owing to the overall factor $\frac{2}{(\dot{A})^2} = 2(n-1)^2(t-t_0)^2$, I becomes divergent when $t \to \infty$ (we assume here that F_{XX} is a positive constant as in the usual ghost condensation model), implying that the mode ψ is certainly dynamical. Second, if we define the square of the mass of ψ by

$$
M_\psi^2 = \frac{n^3}{2n-1}\frac{4K}{3}m^{2n}e^{-2(n-1)A}, \tag{63}
$$

it is also positive for $n > \frac{1}{2}$ and approaches zero in the limit $t \to \infty$ like the tensor modes. The region of $n > \frac{1}{2}$ is consistent with $n < 1$ for the positive acceleration and $n > \frac{-1+\sqrt{33}}{16}$ for the positivity of the square of the graviton mass. Third, as seen easily in (62), we have an unusal time-dependent dispersion relation, so the scalar mode ψ in general breaks the Lorentz invariance. Finally, in the case of $n = 0$, this model reduces to the original ghost condensation one.

Hence, in this section, we have explicitly shown that only the propagating modes around the cosmological backgrounds in this simple massive gravity are the massive tensor modes h_{ij}^{TT} and one scalar mode ψ. As one peculiar feature, the massive scalar mode breaks the Lorentz symmetry manifestly. Even if we have limited ourselves to the simple massive gravity, we believe that this feature is also shared by the most general massive gravity models.

5 The absence of non-unitary mode in the 't Hooft model with ghost condensation

In this section, for completeness, we shall present a proof of the absence of the non-unitary mode in the 't Hooft model with ghost condensation which was considered in section 2.

Now let us start by considering the fluctuations around the unitary gauge (8) for the four scalar fields

$$\phi^a = m x^\mu \delta_\mu^a + \pi^a. \tag{64}$$

Then, X is expanded as

$$\begin{aligned} X &= -m^2 e^{-2A} + e^{-2A}(-m^2 h_{00} + 2m\partial_0\pi_0) \\ &\equiv \bar{X} + e^{-2A}(-m^2 h_{00} + 2m\partial_0\pi_0). \end{aligned} \tag{65}$$

With Eq. (64), the ϕ^i-equations of motion take the form

$$\partial_\mu h^{\mu i} - \frac{1}{2}\partial^i h - \frac{1}{m}\Box\pi^i + 2\dot{A}(h^{0i} - \frac{1}{m}\partial^0\pi^i) = 0. \tag{66}$$

And the ϕ^0-equation reads

$$\partial_0[F_{XX}(\bar{X})(h_{00} - \frac{2}{m}\partial_0\pi_0)] = 0. \tag{67}$$

At this stage, let us note that in the usual ghost condensation models which satisfy the ansatzes (3), one takes, for instance, the potential $F(X)$ to be $F(X) = c_1(X - \bar{X})^2 + c_2$ with some constants $c_1 > 0$ and c_2. Then, it is natural to assume $\partial_0 F_{XX}(\bar{X}) = 0$. With this assumption, the ϕ^0-equation reduces to

$$\partial_0(h_{00} - \frac{2}{m}\partial_0\pi_0) = 0. \tag{68}$$

This equation is easily solved to

$$h_{00} - \frac{2}{m}\partial_0 \pi_0 = C(x^i), \tag{69}$$

where $C(x^i)$ is an integration function depending on x^i but not t. We then find that $C(x^i)$ can be absorbed into the definition of π_0 by redefining

$$\pi_0 \rightarrow \pi_0 + \frac{m}{2} t C(x^i). \tag{70}$$

Thus, we can take $C(x^i) = 0$. Then, π_0 can be expressed in terms of h_{00}, so π_0 is not an independent degree of freedom and can be neglected from the physical spectrum.

The remaining equations of motion which we have to examine are Einstein's equations. At this point, for spatial diffeomorphisms, we take the gauge conditions

$$\pi^i = 0. \tag{71}$$

With the help of the gauge conditions (71) and the ϕ^0-equation (69) with $C(x^i) = 0$, Einstein's equations read

$$\begin{aligned} & \frac{1}{2}[\partial_\rho \partial_\mu h^\rho_\nu + \partial_\rho \partial_\nu h^\rho_\mu - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h - \eta_{\mu\nu}(\partial_\rho \partial_\sigma h^{\rho\sigma} - \Box h)] \\ + \; & \frac{m^2}{2}[2\delta^0_\mu \delta^0_\nu - (1 + h_{00})\eta_{\mu\nu} - h_{\mu\nu}] \\ + \; & \dot{A}[-2\eta_{\mu\nu}(\partial_\rho h^{0\rho} + \frac{1}{2}\partial_0 h) + 2\Gamma^0_{\mu\nu}(h)] \\ = \; & m^2 \delta^i_\mu \delta^j_\nu \delta_{ij} - \frac{m^2}{2}(3 - h_{ii})\eta_{\mu\nu} - \frac{3}{2} m^2 h_{\mu\nu}, \end{aligned} \tag{72}$$

where $\Gamma^0_{\mu\nu}(h)$ denotes the Christoffel symbol in the linear approximation of h.

In order to transform Einstein's equations (72) to the more tractable form, let us consider the following quantity

$$P^\mu = \partial_\nu h^{\nu\mu} - \frac{1}{2}\partial^\mu h + 2\dot{A} h^{0\mu}. \tag{73}$$

With the ϕ^i-equations (66) and the gauge conditions (71), we have

$$P^i = 0. \tag{74}$$

Under the general coordinate transformations (57), the time-like component of P^μ transforms as

$$\delta P^0 = \Box \zeta_0 - 2\dot{A}\partial_0 \zeta_0 - 2m^2 \zeta_0. \tag{75}$$

Thus, using the remaining time-like diffeomorphism, we shall take a gauge [9]

$$P^0 = 0. \tag{76}$$

[9] The same gauge condition was also taken in [10].

Then, a little thought or calculation shows that Einstein's equations reduce to

$$\Box \hat{h}_{\mu\nu} - 2\dot{A}\partial_0 \hat{h}_{\mu\nu} = 2m^2 \hat{h}_{\mu\nu}, \tag{77}$$

where we have defined $\hat{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$.

Notice that the right-hand side of δP^0 in Eq. (75) has the same expression as the equations of motion for $\hat{h}_{\mu\nu}$. Given that $\Box\zeta_0 - 2\dot{A}\partial_0\zeta_0 - 2m^2\zeta_0 = 0$, we obtain $\delta P^0 = 0$. In other words, there remains a residual gauge symmetry associated with such the ζ_0. The existence of the residual symmetry makes it possible to take a gauge

$$h = 0, \tag{78}$$

from which we have the equation

$$\hat{h}_{\mu\nu} = h_{\mu\nu}. \tag{79}$$

Finally, introducing $H_{\mu\nu} \equiv e^{A(t)} h_{\mu\nu}$, it turns out that the whole equations read

$$\begin{aligned}
\Box H_{\mu\nu} - \frac{3}{2}m^2 H_{\mu\nu} &= 0, \\
\partial^\nu H_{\mu\nu} - \dot{A} H_{0\mu} &= 0, \\
H &= 0, \\
e^{-A} H_{00} - \frac{2}{m}\partial_0 \pi_0 &= 0.
\end{aligned} \tag{80}$$

These equations show that the graviton has mass $\sqrt{\frac{3}{2}}m$ and the same 5 degrees of freedom as usual massive graviton modes. Moreover, there is no non-unitary mode since π_0 mode is expressed in terms of H_{00}. In this way, we find that the 't Hooft model with ghost condensation describes a physically plausible massive gravity model in the linearly expanding universe with zero acceleration.

6 Conclusions and Discussion

In this paper, we have shown that the 't Hooft model with ghost condensation is free of non-unitary scalar mode and is a massive gravity model in the linearly expanding cosmological universe with zero acceleration. This proof is rather general and simple. The reason is that in this model the acceleration is vanishing, from which many of equations take tractable expressions. Notice that the situation where there is no acceleration in the Friedmann-Robertson-Walker metric is similar to that where the equation of state is $P = -\frac{1}{3}\rho$ with P and ρ being respectively the pressure and the matter density. This analogy might be useful for a better understanding of this solution.

Furthermore, we have showed that a more general massive gravity model has an interesting class of classical solutions with the property of polynomially expanding cosmological universes

with non-zero acceleration. This class of solutions is classified by a constant n in the potential. Recall that this constant n is not a completely free parameter but receives some restriction from physical conditions. The first requirement that the model should describe a positive acceleration leads to $n < 1$. The second requirement that the massive graviton should not be a tachyon gives us a condition $n > \frac{-1+\sqrt{33}}{16}$ or $n < \frac{-1-\sqrt{33}}{16}$ $(n \neq 1)$. Moreover, the third requirement that the scalar mode should not be a ghost and/or a tachyon provides a final condition $n > \frac{1}{2}$. As a result, there exists the parameter region $\frac{1}{2} < n < 1$, which satisfies three requirements above at the same time.

The existence of non-zero acceleration in the polynomially expanding cosmological universes has made it difficult to prove that this massive gravity is free of the non-unitary mode and the Boulware-Deser instability. In order to clarify the physical propagating modes, we have used the (1+3)-parametrization of the metric fluctuations. Using this parametrization, it has been explicitly shown that the tensor modes become massive, the vector modes are non-dynamical, three of the scalar modes are also non-dynamical and one scalar becomes massive. This scalar is originally non-unitary mode, that is, a ghost, but becomes a normal particle because of the ghost condensation mechanism. However, the dispersion relation is not usual one, so this mode breaks the Lorentz invariance as expected from the ghost condensation scenario.

In most of models which attempt to explain the late-time cosmic acceleration, the acceleration is driven by some exotic matter with negative pressure called dark matter. On the other hand, in the models considered in this paper, the acceleration is driven by the massive graviton and an extra scalar which are originally part of components of the metric tensor $h_{\mu\nu}$. This fact is in sharp contrast to the models which have been proposed so far. To our knowledge, no serious attempts have been made to study the late-time cosmic acceleration this way.

As future's problems, we first wish to construct a Lorentz-invariant massive gravity model in a flat Minkowski space-time such that the model is free of the non-unitary mode since such a model describes a world of QCD. We also wish to examine various phenomenological aspects of the models that we have considered in this paper. It is known that the models lead to interesting phenomenology around a flat Minkowski background [14]-[16], so we think that the models around our cosmological backgrounds also give us new definite predictions for a scenario for inflation and density perturbations.

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